# Zeolitic Diffusivities of Hydrocarbons by the Frequency Response Method

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In order to prove the efficiency of the new frequency response method by actual data, it is applied to propane and ethane over Linde 5A systems, in which considerable discrepancies remain unresolved between the diffusivities determined by the sorption and nuclear magnetic resonance (NMR) methods. The frequency response measurements of the systems reveal that both systems are composed of two types of intracrystalline diffusion processes whose Fickian diffusivities differ from each other four orders of magnitude; the large diffusivity agrees with that obtained from NMR experiments, whereas the small diffusivity corresponds to none of the diffusivities by standard methods. Based on the present results, a mode1 of the zeolitic diffusion which contains two kinds of admolecules, tightly and loosely bound within the cages, is proposed. The frequency response method would be effective therefore to clarify the *whole* aspect of the kinetic phenomena which seems available to characterize zeolite catalysts.  $\circ$  1985 Academic Press, Inc.

crystalline volumes characteristic of zeo- sion coefficients (7) and the method has lites make them ideally suited for shapeintracrystalline diffusion coefficient of a gas is therefore a fundamental quantity to char-<br>acterize the micropores and/or intracrystal-<br>with those previously obtained by the stanacterize the micropores and/or intracrystalline volumes. dard methods.

The diffusion coefficient of a gas has usually been determined by matching experimental sorption curves to appropriate solu-<br>tion of a diffusion equation  $(2, 3)$ . The *Apparatus*. The FR apparatus was altion of a diffusion equation  $(2, 3)$ . The NMR-pulsed field gradient technique, on most the same as that described elsewhere the other hand, has been shown to be effec-  $(9)$ ; the mean volume of the sorption chamthe other hand, has been shown to be effective to determine the coefficients of hydro- ber,  $V_e$ , was 1.145 and 1.120 dm<sup>3</sup> when the carbons (4, 5). However, large discrepan- sorbent was maintained at ( $T_e =$ ) 273 and carbons (4, 5). However, large discrepan- sorbent was maintained at  $(T_e =)$  273 and cies exist between the results by the two 366 K, respectively; the relative amplitude cies exist between the results by the two 366 K, respectively; the relative amplitude methods  $(4, 6)$ . Though the discrepancies of the sinusoidal volume variation, v, was methods (4, 6). Though the discrepancies of the sinusoidal volume variation, v, was have been reduced in some cases for conju-  $2.20 \times 10^{-2}$  (at 273 K) or  $2.23 \times 10^{-2}$  (at 366 have been reduced in some cases for conjugated NMR and sorption experiments on K) in standard runs.<br>the same (homemade) samples of large Sorbent. Commercial synthetic 5A zeothe same (homemade) samples of large crystal size, they remain unresolved for lites, Linde 5A (binder free), were used as commercial molecular sieves such as Linde sorbent. The powders pelletized were de-5A of smaller crystal size (6). hydrated at 383 K for 10 h. The temperature

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INTRODUCTION In a previous paper one of the authors proposed a frequency response (FR)<br>method of determination of zeolitic diffu-The small uniform pores and large intra- method of determination of zeolitic diffu-<br>vstalline volumes characteristic of zeo- sion coefficients (7) and the method has selective catalyst support (*I*). The over Linde 5A systems ( $8$ ). It seems of in-<br>intracrystalline diffusion coefficient of a gas terest to apply it further to a hydrocarbon/

was raised to 673 K and evacuated at that i To whom all correspondence should be addressed. level for at least 30 h. The amount of the



FIG. 1. Frequency response of the propane/Linde 5A system at  $T_e = 273$  K and  $P_e = 9$  Torr vs the angular velocity  $\omega$ : ( $\square$  and  $\square$ ) in-phase component of  $(v/p)$ cos  $\varphi - 1$ ; ( $\circ$  and  $\Phi$ ) out-of-phase component of  $(v/p)\sin \varphi$ ; where (Cl, O) and (1, **O**) are the results measured at  $P_e = 8.8$  and 9.2 Torr, respectively. The heavy solid curves are calculated from Eqs. (4) and (5): the light solid curves indicate contribution of the first terms and the shaded areas contribution of the second terms.

sorbent after these dehydrations was 9.87 g.

Sorbate. Tank propane (98 mol%; impurities were 1.3 mol% of  $C_2H_6$ , 0.7 mol% of i- $C_4H_{10}$ , and 0.4 mol% of  $C_4H_{10}$ ) or ethane  $($ >99.9 mol%) was used as the sorbate.

### RESULTS

In-phase and out-of-phase components,  $(v/p)$ cos  $\varphi$  - 1 and  $(v/p)$ sin  $\varphi$ , respectively, are plotted versus  $\omega$  in Fig. 1, where p denotes the relative amplitude of the pressure variation induced,  $\varphi$  is the phase lag between the volume and pressure variations, and  $\omega$  is the angular velocity of the sinusoidal waves. The periods of the oscillations corresponding to the largest and smallest  $\omega$ 's in Fig. 1 were 1.2 s and 1.5 h, respectively. Every component is the average value of at least four periods.

According to the theoretical procedure (7), the components may be described by

$$
(v/p)\cos\varphi-1=\sum_{n}K_{n}\overline{\delta}_{3c}^{(n)}\qquad(1)
$$

$$
(v/p)\sin\,\varphi\,=\,\sum_n\,K_n\overline{\delta}^{(n)}_{3s},\qquad(2)
$$

where  $\bar{\delta}_{3c}$  and  $\bar{\delta}_{3s}$  denote the characteristic functions modified by a crystal-size distribution. It is found that, on the basis of the results obtained from the Kr/Linde 5A system  $(8)$ , the functions can explicitly be given in the present case by

$$
\overline{\delta}_{3j}^{(n)} = \frac{1}{(2\pi)^{1/2}\sigma} \int_0^\infty \delta_{3j} \left(\frac{D_n}{a_x^2}\right)
$$

$$
\exp\left\{-\frac{(a_x - a_m)^2}{2\sigma^2}\right\} da_x
$$

$$
(j = c \text{ or } s), \quad (3)
$$

where the standard deviation relative to the mean radius  $a_m$ ,  $\sigma/a_m$ , was 0.5 (various defects in the crystal which have been observed with high-resolution electron microscopy (10) could have effectively been considered in this value besides the apparent value of 0.3-0.4 by a photomicrographic technique  $(11)$ ); the explicit formulas of the characteristic functions,  $\delta_{3c}$  and  $\delta_{3s}$ , based on an isotropic sphere model are given elsewhere  $(7, 8)$ . It should be noted that  $\delta_{3s}$  (and then  $\delta_{3s}$ ) as a function of  $\omega$  has a single peak and that  $\delta_{3c}$  (and then  $\delta_{3c}$ ) tends to unity as  $\omega \rightarrow 0$ .

Since the results in Fig. 1 evidently reveal more than one peak in the out-of-phase components, two kinds of diffusion processes are assumed here. The heavy solid curves represent the theoretical results calculated from

$$
(v/p)\cos\varphi-1=K_1\overline{\delta}_{3c}^{(1)}+K_2\overline{\delta}_{3c}^{(2)}\qquad(4)
$$

$$
(v/p)\sin \varphi = K_1 \overline{\delta}_{3s}^{(1)} + K_2 \overline{\delta}_{3s}^{(2)}, \quad (5)
$$

where the values of the four parameters,  $K_1$ ,  $K_2$ ,  $\tilde{D}_1(= D_1/a_m^2)$ , and  $\tilde{D}_2(= D_2/a_m^2)$ , given in Table 1, were determined as follows. The asymptotic value of the in-phase component in a lower  $\omega$  region,  $K_1 + K_2$ , must be proportional to the gradient of the equilibrium isotherm (7):

$$
K_1 + K_2 = \frac{RT_0}{V_e} \frac{d}{dP_e} (B_1 + B_2), \quad (6)
$$

where  $(RT_0/V_e)$  is the conversion factor and  $B_n$  denotes the amount of admolecules to



Exp. No.	$P_{e}$ Torr	$M^a$	$N^b$	$\log \omega^c$	$\tilde{D}_1$ $min-1$	D <sub>2</sub> $min^{-1}$	$K_1$		$K_2$ $\Delta_c \times 10^{2d}$ $\Delta_s \times 10^{2e}$	
(Propane)										
		$2.8$ 2.5		$11 -0.3 - 2.4$ 7.54	$(\pm 0.05)$	$5.5(\pm 0.5) \times 10^{-5}$	0.98 $(\pm 0.03)$	12.0 $(\pm 1)$	1.42	2.34
$\mathbf{2}$		$5.4$ 2.9		$13 -0.8 - 2.5$ 5.85	(±0.05)	$8.0(\pm 0.5) \times 10^{-5}$	0.75 $(\pm 0.03)$	6.5 $(\pm 0.5)$	2.86	3.49
3 $\overline{\mathbf{4}}$		$8.8 \quad 3.2$ $9.2 \quad 3.2$	21	$-1.2 - 2.5$	5.35 $(\pm 0.03)$	$3.0(\pm 0.5) \times 10^{-4}$	0.85 $(\pm 0.03)$	4.2 $(\pm 0.3)$	2.99	2.31
5 (Ethane)	16.3	4.1	11	$0.1 - 2.5$	4.85 $(\pm 0.05)$	$1.2(\pm 0.5) \times 10^{-3}$	0.74 $(\pm 0.03)$	1.2 $(\pm 0.3)$	2.64	2.36
6		$5.2\quad 0.6$	11	$-1.1 - 2.5$ 2.10	$(\pm 0.05)$	$3.8(\pm 0.5) \times 10^{-3}$	4.74 $(\pm 0.03)$	6.25 $(\pm 0.2)$	5.75	15.0

Experimental Conditions and Concluded Values of the Parameters in Eqs. (4) and (5)

<sup>a</sup> The number of molecules in a cage.

 $<sup>b</sup>$  The number of scans at different  $\omega$ .</sup>

 $c$  The range of  $\omega$  covered.

<sup>d</sup> The mean deviation of in-phase component defined by  $\Delta_c = (1/N) \sum_i |\text{obsd} - \text{calcd}|\$ .

<sup>e</sup> The mean deviation of out-of-phase component defined by  $\Delta_s = (1/N) \sum_i |\text{obsd} - \text{calcd}|\$ .

which  $K_n$  and  $D_n$  are attributable. Therefore, the sum of  $K_1 + K_2$  can be predicted from the gradient of the isotherm. On the other hand, the height of the out-of-phase component in the higher  $\omega$  region in Fig. 1 would approximately be 0.27  $K_1$  (8). The most probable values of the parameters in Table 1 were determined after repeated calculation by a computer for various choices of them around the initial values thus estimated. Contribution of the first terms is represented by the light solid curves and that of the second terms is indicated by the shaded areas.

The experiments were repeated at different equilibrium pressures. The values of the parameters determined by the computer simulation are summarized in Table 1 and plotted in Fig. 2 as functions of the equilibrium pressure.

The nearly constant  $K_1$  means that  $B_1$  is almost proportional to  $P_e$  so that  $B_1$  species would be loosely bound. On the other hand, the sudden decrease in  $K_2$  with increasing  $P_e$  indicates the saturation of  $B_2$  at the higher pressure or tightly bound  $B_2$  species. The ratio  $B_1/B_2$  could be derived from the pressure dependence and found to be 0.1 at  $P_e$  = 9 Torr (1 Torr = 133.3 Pa) of Fig. 1, when total amounts of admolecules,  $B_1$  +  $B<sub>2</sub>$ , were 1.9 mmol/g or 3.2 molecules/cage.

In contrast with the case of Kr or Xe/



FIG. 2. Plots of the parameters in Table 1 as functions of  $P_e$ : all circles refer to  $n = 1$ ; all squares to  $n =$ 2. Solid symbols are the corrected diffusivities derived from the parameters.



FIG. 3. Frequency response of the ethane/Linde 5A system at  $T_e = 273$  K and  $P_e = 5.2$  Torr vs the angular velocity  $\omega$ : ( $\square$ ,  $\square$ ) with  $v = 2.23 \times 10^{-2}$ ; ( $\square$ ,  $\spadesuit$ ) with v  $= 5.03 \times 10^{-2}$ . Notation is that of Fig. 1.

Linde 5A (8), two kinds of admolecules are required in the data analysis. In order to confirm the unexpected results, additional experiments with ethane of adequate purity instead of propane were carried out. The results are plotted in Fig. 3; the solid curves represent the computed results with the parameters given in Table 1. It is noted that  $K_1$ is comparable to  $K_2$  in this case in contrast with the case of propane  $(K_1 \ll K_2)$ . In the course of the experiments,  $v$  was increased from 2.23  $\times$  10<sup>-2</sup> to 5.03  $\times$  10<sup>-2</sup>. However, the FR data were practically unaltered as shown in Fig. 3. It may be concluded therefore that effects of both heat and mass transfer resistance  $(12)$  are negligible in the present work.

Typical data of propane observed at a higher equilibrium temperature, 366 K, are shown in Fig. 4, in which total amount of admolecules was 0.27 mmol/g or 0.4 molecules/cage; the asymptotic value of the inphase component obtained from the gradient of the isotherm was 4.0. Evidently, it is impossible for them to be described by Eqs. (1) and (2) because the in-phase components are smaller than the out-of-phase components in the higher  $\omega$  region. This means that Fick's law is inadequate in this case. It should be noted however that there appear two peaks of comparable heights in the out-of-phase components and that the

amount of  $B_1$  species at 366 K would be larger than that at 273 K at an equilibrium pressure around 5 Torr because the first peak was higher than that of Fig. 1.

### DISCUSSION

The Fickian diffusivities  $D_1$  and  $D_2$  can be evaluated from  $\tilde{D}_1$  and  $\tilde{D}_2$  provided  $a_m = 1.8$  $\mu$ m (11) and the corrected diffusivities based on a chemical potential driving force  $(13)$ ,  $D_{01}$  and  $D_{02}$ , could be given by

$$
D_n = D_{0n} \left( \frac{\partial \ln P_e}{\partial \ln B_n} \right)_T \tag{7}
$$

provided the *individual* isotherm  $B_n(P_e)$  are derived from the  $P_e$  dependence of  $K_n$ . The results are shown by the solid symbols in Fig. 2. They are compared further with the values determined by the NMR and sorption techniques (6) in Fig. 5. The fast diffusivity would correspond to the results by the NMR method because it agrees with the value on the dashed line extrapolated (identical activation energy was obtained from diffusivities by NMR and sorption experiments in the case of butane (6)).

Based on the experimental results, a model of the diffusion shown in Fig. 6 could be derived. The depth of the potential wells for  $B_2$  species of propane is ca. 35 kJ/mol; this is the heat of absorption determined from the equilibrium isotherms by making use of Clausius-Clapeyron equation. The potential barrier for crossing the windows



FIG. 4. Frequency response of the propane/Linde 5A system at  $T_e = 366$  K and  $P_e = 8.9$  Torr vs the angular velocity  $\omega$ . Notation is that of Fig. 1 but the smoothed curves are temporarily drawn.



FIG. 5. Diffusivities of propane in zeolite 5A determined by different methods vs the equilibrium temperature  $T<sub>e</sub>$ : the top and bottom of the rectangles indicate the maximum and minimum of the diffusivities by the FR method; (O) the diffusivities in Linde 5A by the sorption method (6); ( $\Delta$ ,  $\blacktriangle$ ) and ( $\blacklozenge$ ,  $\blacklozenge$ ) are the diffusivities in zeolite SA of large crystal sizes determined by the NMR and sorption methods, respectively (6).

is ca. 15 kJ/mol; this is the apparent activation energy of the solid and dashed lines in Fig. 5.

According to the model, a rough estimation of the temperature dependence of  $B_1/$  $B<sub>2</sub>$  is possible:

$$
\frac{B_1}{B_2} = \frac{q_1}{q_2} \exp \left\{-\left(\frac{35-15}{8.3\times10^{-3}}\right) \middle/ T_e\right\},\quad (8)
$$

where  $q_n$  denotes the partition function. Since  $B_1/B_2$  was 0.1 at 273 K,  $q_1/q_2$  is expected to be about 700. On the assumption



FIG. 6. Model of the zeolitic diffusion of propane: the upper side is schematic diagram of the diffusion in the micropores; the lower is the potential energy profiles for  $B_1$  and  $B_2$  species of propane absorbed. The depth of the potential wells for  $B_2$  is ca. 35 kJ/mol and the barrier for  $B_1$  to cross the window is ca. 15 kJ/mol.

of  $q_1/q_2$  being constant, one finds that  $B_1/B_2$  $=$  1 and 4 at 366 and 473 K, respectively. Consequently,  $B_1$  species are probably dominant at the higher temperatures of Fig. 5 where the NMR experiments (and also the sorption experiments of large crystal sizes) were performed; the results by both NMR  $(D<sub>s</sub>)$  and sorption techniques would necessarily correspond to  $D_{01}$ . The failure of the curve fitting the FR data at 366 K in Fig. 4 is perhaps due to the comparable amounts of  $B_1$  and  $B_2$  species or fast transition between them at the intermediate temperature.

The discrepancies between the values obtained from the FR and sorption experiments at a lower temperature could be explained as follows. The sorption measurements were usually performed at higher pressures, because reliable sorption diffusivities could not be obtained at low sorbate concentrations  $(6)$ . Since  $B_2$  species is expected to be nearly saturated at the high pressure, the amount of  $B_1$  species alone would be affected by a (small) step change in the pressure. Consequently,  $B_2$ species could not have been detected by the sorption method. On the other hand, the (small) step change is enormously larger than the change induced by the volume variation in the FR experiments and therefore the gradient of the concentration of sorbate would be considerably large at an entrance of the micropores. Dependence of the flux of sorbate  $j$  at the entrance upon the gradient  $d c/dx$  is shown in Fig. 7; in the



FIG. 7. Flux of molecules  $j$  at an entrance of micropores vs the gradient of concentration of sorbate at the entrance  $dc/dx$ .

small gradient region, *j* may be proportional to the gradient according to Fick's first law, while  $j$  would asymptotically reach the limit  $j_{\text{max}}$  with increasing gradient because of the finite size of the entrance. Consequently, the diffusivity by the sorption method should unavoidably be underestimated, though the activation energy would be unaltered irrespective of the saturation of the flux because only the potential barrier controls the diffusion process.

The remarkable increase in  $D_2$  with increasing amounts of admolecules (from 2.5 to 4.1 molecules/cage) is probably attributed to repulsive forces between  $B_2$  species or a finite number of minima in the potential energy; the saturation concentration at 273 K is expected to be 4.6 molecules/cage  $(13).$ 

In the case of Xe molecules at 273 K  $(8)$ , whose cross section is almost the same as that of n-paraffins, the FR data led to only one diffusion constant, while those of both propane and ethane reveal the two types. The fact supports the conclusion that the two types of diffusion processes can be expressed in terms of the two kinds of admolecules rather than inter- and intracrystalline diffusions. Potential energy profiles for a xenon atom in an idealized 5A zeolite cavity have been calculated and two potential minima of 33 and 25 kJ/mol are concluded  $(14).$ 

Since intracrystalline volumes as well as micropores would affect the activity and/or selectivity of zeolite catalyst, it seems of interest to study the effects of various modifications of the catalyst upon each of the

diffusion processes individually; modifications such as dehydration, cation exchange, preadsorption of a gas, deactivation by a reaction, and so on. The present method may be regarded as a *probe* for the micropores by means of various gases.

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